Besvarelser til Lineær Algebra Reeksamen - 17. Februar 2017

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Bemærk, at der kan være sneget sig fejl ind.
Kontakt mig endelig, hvis du skulle falde over en sådan.
Dette dokument har udelukkende til opgave at forklare,
hvordan man kommer frem til facit i de enkelte opgaver.
Der er altså ikke afkrydsningsfelter, der er kun facit og tilhørende udregninger.
Udregningerne er meget udpenslede, så de fleste kan være med.

Indhold

Problem 1

Consider the matrices

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

and answer the following questions.

1. Is A in row echelon form?

No, all zero rows must be at the bottom of the matrix.

2. Is B in row echelon form?

Yes, there is no non-pivot row before a pivot row and below each pivot entry there are zeros.

3. Is A in reduced row echelon form?

No, if it is not in row echelon form it can not be in reduced row echelon form!

4. Is B in reduced row echelon form?

No, we need 0 both below AND over each pivot entry.

5. Can B be obtained from A by elementary row operations?

No, there is only zeros in the first column of A which is not the case of B.
Another way of seeing this is noticing, that x_1 is a free variable of A, but this is not the case of B.

6. Can A be obtained from B by elementary row operations?

No, if you can not go one way with elementary row operations, then you can not go the other way.

Problem 2

Let $A = [\mathbf{a}_1 \;\; \mathbf{a}_2]$ be a matrix with 4 rows and let $B = [\mathbf{b}_1 \;\; \mathbf{b}_2 \;\; \mathbf{b}_3 \;\; \mathbf{b}_4 \;\; \mathbf{b}_5]$ be such that $C = AB$ is defined.

1. How many rows are there in the matrix B?

We know that A is a 4×2-matrix. Hence for AB to be defined B must be a 2×5-matrix (we already know that B has 5 columns). We conclude that B has 2 rows.

2. How many rows are there in the matrix C?

Since A is the leading matrix C must have the same amount of rows as A meaning 4.

Problem 3

Let $A = \begin{bmatrix} 4 & 3 \\ 3 & -4 \end{bmatrix}$ and let $B = \frac{1}{5}A$.

1a) A is an orthogonal matrix.

False, the columns does not have length 1.

1b) B is an orthogonal matrix.

True, because:

$$\frac{1}{5}\begin{bmatrix} 4 \\ 3 \end{bmatrix} \cdot \frac{1}{5}\begin{bmatrix} 3 \\ -4 \end{bmatrix} = \frac{1}{25}(4 \cdot 3 + 3 \cdot (-4)) = 0$$

Hence the columns are orthogonal. They have the same length, but is the length 1?

$$\frac{1}{5}\sqrt{4^2 + 3^2} = \frac{1}{5}\sqrt{25} = \frac{1}{5} \cdot 5 = 1.$$

Yes, the length is 1, therefore the matrix is orthogonal.

1c) A is a symmetric matrix.

True. For A to be symmetric, we must have $A = A^\top$ which is the case.

1d) $B^{-1} = -B$.

False. We know that B is an orthogonal matrix. That means $B^{-1} = B^\top$. Furthermore we know that A is symmetric, which means B is symmetric and therefore $B = B^\top$. But then $B = B^\top = B^{-1}$. But $B = -B$ only if B is a matrix consisting of only zeros - which is not the case.

1e) $B^{-1} = B$.

True - see 1d) for reasoning.

2. What is the determinant of B?

We have

$$\det B = \det\left(\frac{1}{5}A\right) = \left(\frac{1}{5}\right)^2 \det A = \frac{1}{25}(4\cdot(-4) - 3\cdot 3) = \frac{1}{25}(-16-9) = \frac{-25}{25} = -1.$$

3. $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be an eigenvector of B with eigenvalue 1. What is the value of x_1 if $x_2 = 1$.

Eigenvectors and their responding eigenvalues should satisfy the equation:

$$B\mathbf{x} = \lambda\mathbf{x}.$$

Let us find the left hand side of the equation first:

$$B\mathbf{x} = \frac{1}{5}\begin{bmatrix} 4 & 3 \\ 3 & -4 \end{bmatrix}\begin{bmatrix} x_1 \\ 1 \end{bmatrix} = \frac{1}{5}\begin{bmatrix} 4x_1 + 3 \\ 3x_1 - 4 \end{bmatrix}.$$

Since $\lambda = 1$ and $\mathbf{x} = \begin{bmatrix} x_1 \\ 1 \end{bmatrix}$ we see that

$$\frac{1}{5}\begin{bmatrix} 4x_1 + 3 \\ 3x_1 - 4 \end{bmatrix} = \begin{bmatrix} x_1 \\ 1 \end{bmatrix}.$$

There is only one unknown variable, x_1, so only one equation is needed. We get:

$$\frac{1}{5}(4x_1 + 3) = x_1 \Leftrightarrow 4x_1 + 3 = 5x_1 \Leftrightarrow x_1 = 3.$$

Problem 4

Let $A = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & 4 & -1 \\ 1 & 1 & 2 & 1 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 0 \\ 3 \\ 4 \end{bmatrix}$. The augmented matrix $\begin{bmatrix} 1 & -1 & 2 & 1 & 0 \\ 2 & 1 & 4 & -1 & 3 \\ 1 & 1 & 2 & 1 & 4 \end{bmatrix}$ has the following reduced row echelon form

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}.$$

1a) Column 1 is a pivot column.

True, because it is the leading digit of the row and there are zeros below.

1b) Column 2 is a pivot column.

True, because it is the leading digit of the row and there are zeros below.

1c) Column 3 is a pivot column.

False, because it is not the leading digit of the row.

1d) Column 4 is a pivot column.

True, because it is the leading digit of the row and there are zeros below.

2. What is the rank of A?

The rank is 3 because A has 3 pivot entries.

3. What is the nullity of A?

Since A has rank 3 and only consists of 4 columns the only $4 - 3 = 1$ dimension is left over for the nullity of A.

4. Let x be a solution of $A\mathbf{x} = \mathbf{b}$. What is x_2?

By finding the pivot entry of the second column of

$$\begin{bmatrix} 1 & 0 & 2 & 0 & 1 \\ 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

we see that x_2 should be 2 (read the last entry of the row, that x_2 is pivot entry for).

Problem 5

Let $A = \begin{bmatrix} 2 & 3 & -5 \\ 4 & 5 & -3 \\ 1 & -3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & 1 \\ 4 & -2 & 1 \\ 5 & 2 & 2 \end{bmatrix}$.
Let $C = AB$. What is the number c_{13}?

Answer:

By the definition of matrix product we can just multiply the first row of A with the third row of B. We have:

$$\begin{bmatrix} 2 & 3 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} = 2 \cdot 1 + 3 \cdot 1 + (-5) \cdot 2 = 2 + 3 - 10 = -5.$$

Problem 6

Let $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}$ and let $W = \text{span}\{\mathbf{v}\}$. Let $\mathbf{u} = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 1 \end{bmatrix}$ and let $\mathbf{w}$ be the orthogonal projection of $\mathbf{u}$ on W.

1. What is the third component of w (i.e. w_3)?

We use the formula

$$\mathbf{w} = \frac{\mathbf{u} \cdot \mathbf{v}}{\|\mathbf{v}\|^2}\mathbf{v}.$$

We get

$$\mathbf{w} = \frac{4 \cdot 1 + 3 \cdot 1 + 2 \cdot (-1) + 1 \cdot (-1)}{1^2 + 1^2 + (-1)^2 + (-1)^2} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \frac{4 + 3 - 2 - 1}{1 + 1 + 1 + 1} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \frac{4}{4} \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ -1 \end{bmatrix}.$$

The third value of $\mathbf{w}$ is then -1. (Strictly speaking there is no reason to do the calculations for the entire vector, but whoever is reading this probably would have gotten confused if I let the other values out).

2. Let z be the orthogonal projection of u on $W^{\perp}$. What is the third component of z (i.e.)?

We know that $\mathbf{u} = \mathbf{w} + \mathbf{z}$ hence $\mathbf{z} = \mathbf{u} - \mathbf{w}$ and therefore $\mathbf{z}_3 = \mathbf{u}_3 - \mathbf{w}_3$ (we only need the third component of each vector):

$$\mathbf{z}_3 = 2 - (-1) = 3.$$

3. What is the dimension of $W^{\perp}$?

Since the vectors belong to $\mathbb{R}^4$ and $W = \text{span}\{\mathbf{v}\}$ (W is only spanned by 1 vector, meaning it has only dimension 1) we know that $\dim\left(W^{\perp}\right) = 4 - 1 = 3$.

Problem 7

Let A and B be 3×3 matrices with $\det A = 5$ and $\det B = 3$.

1. What is $\det(-2A)$?

We will use the a property of determinants. We have:

$$\det(-2A) = (-2)^3 \det A = -8 \det A = -8 \cdot 5 = -40.$$

Reminder of property: The determinant of $\det(kC)$ where k is a scalar and C is a $n \times n$-matrix is given by $\det(kC) = k^n \det(C)$.

2. What is $\det AB^\top$?

We will use two other properties. The first property is $\det(CD) = \det C \det D$ for C and D being $n \times n$-matrices. The second is $\det(C^\top) = \det(C)$. We have:

$$\det(AB^\top) = \det(A)\det(B^\top) = \det(A)\det(B) = 5 \cdot 3 = 15.$$

3. What is $\det AB^{-1}$?

We will use the matrix multiplication property of the determinant as in 2. Furthermore we will use the property, that $\det(C^{-1}) = \det(C)^{-1} = \frac{1}{\det(C)}$. We have:

$$\det(AB^{-1}) = \det(A)\det(B^{-1}) = \det(A)\det(B)^{-1}) = \frac{\det A}{\det B} = \frac{5}{3}.$$

Problem 8

The characteristic polynomial of $A = \begin{bmatrix} -8 & 4 & -2 & 10 \\ 0 & -1 & 0 & 0 \\ 2 & -2 & 0 & -2 \\ -7 & 4 & -2 & 9 \end{bmatrix}$ is $t(t-2)(t+1)^2$.

1. Which one of the following is an eigenvalue of A?

It is -1 because eigenvalues are the values of t that makes $t(t-2)(t+1)^2 = 0$.

2. Which one of the following is an eigenvector of A?

$$(a)\ \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},\quad (b)\ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix},\quad (c)\ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix},\quad (d)\ \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

The zero-vector (a) can not be an eigenvector per definition of eigenvectors (all *non-zero* vectors, that satisfy $A\mathbf{v} = \lambda\mathbf{v}$).

We multiply A with each vector, (b), (c) and (d) independently. We have for (b):

$$A \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 & 4 & -2 & 10 \\ 0 & -1 & 0 & 0 \\ 2 & -2 & 0 & -2 \\ -7 & 4 & -2 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 \cdot 1 + 4 \cdot 1 \\ -1 \cdot 1 + 0 \cdot 1 \\ 2 \cdot 1 + (-2) \cdot 1 \\ -7 \cdot 1 + 4 \cdot 1 \end{bmatrix} = \begin{bmatrix} -8+4 \\ -1+0 \\ 2-2 \\ -7+4 \end{bmatrix} = \begin{bmatrix} -4 \\ -1 \\ 0 \\ -3 \end{bmatrix}.$$

(b) can not be an eigenvector, because we can not find a scalar λ such that:

$$\begin{bmatrix} -4 \\ -1 \\ 0 \\ -3 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}.$$

Similar calculations are done for (c):

$$A \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 & 4 & -2 & 10 \\ 0 & -1 & 0 & 0 \\ 2 & -2 & 0 & -2 \\ -7 & 4 & -2 & 9 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -8 \cdot 1 + (-2) \cdot 1 \\ 0 \cdot 1 + 0 \cdot 1 \\ 2 \cdot 1 + 0 \cdot 1 \\ -7 \cdot 1 + (-2) \cdot 1 \end{bmatrix} = \begin{bmatrix} -8-2 \\ 0+0 \\ 2+0 \\ -7-2 \end{bmatrix} = \begin{bmatrix} -10 \\ 0 \\ 2 \\ -9 \end{bmatrix}.$$

(c) can not be an eigenvector, because we can not find a scalar λ such that:

$$\begin{bmatrix} -10 \\ 0 \\ 2 \\ -9 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

Similar calculations are done for (c):

$$A\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \begin{bmatrix}-8 & 4 & -2 & 10\\0 & -1 & 0 & 0\\2 & -2 & 0 & -2\\-7 & 4 & -2 & 9\end{bmatrix}\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = \begin{bmatrix}-8\cdot 1+10\cdot 1\\0\cdot 1+0\cdot 1\\2\cdot 1+(-2)\cdot 1\\-7\cdot 1+9\cdot 1\end{bmatrix} = \begin{bmatrix}-8+10\\0+0\\2-2\\-7+9\end{bmatrix} = \begin{bmatrix}2\\0\\0\\2\end{bmatrix} = 2\begin{bmatrix}1\\0\\0\\1\end{bmatrix}.$$

Since

$$A\begin{bmatrix}1\\0\\0\\1\end{bmatrix} = 2\begin{bmatrix}1\\0\\0\\1\end{bmatrix}$$

we have that (d) is an eigenvector with the corresponding eigenvalue 2.

3. Is A invertible?

No! Why? Because A is only invertible if 0 is NOT an eigenvalue of A - and in this case 0 is an eigenvalue because for $t = 0$ the characteristic polynomial $t(t-2)(t+1)^2$ will equal 0.

Problem 9

Let

$$A = \begin{bmatrix} 2 & 4 & 1 & 2 \\ 2 & 4 & 1 & 3 \\ 0 & 1 & 0 & 0 \\ 1 & 2 & 3 & 4 \end{bmatrix}.$$

What is the determinant of A?

Answer:

Using the cofactering method around the third 3 row, we get:

$$\det(A) = (-1)^{3+1}\cdot 0\cdot \det A_{31} + (-1)^{3+2}\cdot 1\cdot \det A_{32} + (-1)^{3+3}\cdot 0\cdot \det A_{33} + (-1)^{3+4}\cdot 0\cdot \det A_{34} = -\det A_{32}$$

where $A_{32} = \begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ is the submatrix of A where third row and second column is removed. Using the cofactering method on this matrix around (just picking a random row) the first row we obtain the determinant for this:

$$\begin{aligned}
\det A_{32} &= \det\left(\begin{bmatrix} 2 & 1 & 2 \\ 2 & 1 & 3 \\ 1 & 3 & 4 \end{bmatrix}\right) \\
&= (-1)^{1+1}\cdot 2\cdot \det\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} + (-1)^{1+2}\cdot 1\cdot \det\begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix} + (-1)^{1+3}\cdot 2\cdot \det\begin{bmatrix} 2 & 1 \\ 1 & 3 \end{bmatrix} \\
&= 2(1\cdot 4 - 3\cdot 3) - 1(2\cdot 4 - 3\cdot 1) + 2(2\cdot 3 - 1\cdot 1) \\
&= 2(4-9) - (8-3) + 2(6-1) = 2\cdot(-5) - 5 + 2\cdot 5 = -5.
\end{aligned}$$

So $\det A_{32} = -5$ and since $A = -\det A_{32}$ we have that $A = -(-5) = 5$.

Problem 10

What is the number of solutions of the following system of linear equations

$$\begin{aligned} x_1 - x_2 + x_4 &= 0 \\ x_1 - x_2 + x_3 + x_4 &= 0 \\ -x_1 + x_2 - x_4 &= 1 \end{aligned}$$

Answer:

We can setup a corresponding matrix for the system of linear equations and reduce it to echelon form:

$$\begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 1 & -1 & 1 & 1 & 0 \\ -1 & 1 & 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

Since the last row has a pivot entry in the last column the system is inconsistent meaning there is 0 solutions.

Problem 11

Let T be the linear transformation with standard matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix}$.
$\mathcal{B} = \{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \}$ is a basis for $\mathbb{R}^2$.
What is the matrix representation of T with respect to $\mathcal{B}$, denoted by $[T]_{\mathcal{B}}$?

Answer:

We will use the formula

$$[T]_{\mathcal{B}} = B^{-1}AB,$$

where $B = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ The determinant of B is:

$$\det B = 1 \cdot 1 - (-2) \cdot 0 = 1,$$

which gives us the inverse of B:

$$B^{-1} = \frac{1}{\det B} \begin{bmatrix} 1 & -(-2) \\ -0 & 1 \end{bmatrix} = \frac{1}{1} \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}.$$

We can now use the formula and get:

$$\begin{aligned} [T]_{\mathcal{B}} &= B^{-1}AB \\ &= \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 3+8 & 2+2 \\ 0+4 & 0+1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 11 & 4 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 11+0 & -22+4 \\ 4+0 & -8+1 \end{bmatrix} \\ &= \begin{bmatrix} 11 & -18 \\ 4 & -7 \end{bmatrix}. \end{aligned}$$

Problem 12

Let $T\colon \mathbb{R}^n \to \mathbb{R}^m$ be the linear transformation with the standard matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}.$$

1. What is the value of n?

The question is "What is the size of a vector $\mathbf{v}$ for $A\mathbf{v}$ to be defined?- The only way we can multiply A with $\mathbf{v}$ like that is for $\mathbf{v}$ to be of dimension 3. Hence $n = 3$.

2. What is the value of m?

The question is "What size does the resulting vector of $A\mathbf{v}$ have?- The resulting vector has the same amount of rows as A hence $m = 5$.

3. What is the rank of A?

We will reduce A to echelon form:

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

Since the echelon form of A has 3 pivot columns the rank of A is 3.

4. What is the dimension of the null space of T?

There are no non-pivot columns in A, hence the null space has no dimension (i.e. the dimension of the null space of T is 0).

5. Is T one-to-one?

Is T injective? Yes, T is injective whenever all columns are pivot entries (i.e. when the null space has dimension 0).

6. Is T onto?

Is T surjective? No, for T to be surjective there must be a pivot entry in all rows of A. This can not be true when there are more rows than columns.

Problem 13

Let $A = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$ and let $\mathbf{b} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$ and $\mathbf{c} = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$.

1. Is b contained in $\operatorname{Col} A$?

We have to see if the system $A\mathbf{x} = \mathbf{b}$ is consistent:

$$\begin{bmatrix} 1 & 1 & 2 & -1 & 3 \\ 2 & -1 & 1 & 1 & 3 \\ 1 & 0 & 1 & 0 & 2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & -1 & 3 \\ 0 & -3 & -3 & 3 & -3 \\ 0 & -1 & -1 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 2 & -1 & 3 \\ 0 & -3 & -3 & 3 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

We know have the system on echelon form hence the system is consistent since there is no pivot in the last column. Therefore **b** is contained in $\operatorname{Col} A$.

2. Is c contained in $\operatorname{Col} A$?

No, because the dimensions of **c** does not fit the dimension of the column vectors in A.

3. Is b contained in $\operatorname{Null} A$?

No, since **b** lies in $\operatorname{Col} A$.

4. Is c contained in the $\operatorname{Null} A$?

We have to multiply A with **c** and see if we get the zero vector:

$$A\mathbf{c} = \begin{bmatrix} 1 & 1 & 2 & -1 \\ 2 & -1 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \cdot (-1) + 1 \cdot 0 + 2 \cdot 1 + (-1) \cdot 1 \\ 2 \cdot (-1) + (-1) \cdot 0 + 1 \cdot 1 + 1 \cdot 1 \\ 1 \cdot (-1) + 0 \cdot 0 + 1 \cdot 1 + 0 \cdot 1 \end{bmatrix} = \begin{bmatrix} -1 + 0 + 2 - 1 \\ -2 + 0 + 1 + 1 \\ -1 + 0 + 1 + 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

Hence **c** is in $\operatorname{Null} A$.

Problem 14

The following commands are entered in the MATLAB Command Window:
» A = [1 1 1 1; 1 2 3 4; 0 1 0 1; 0 0 0 1];
» b = [1; 0; 1; 2];
» T = [A b];

1. What is the size of the matrix T?

T has the size of A + an extra column (b). Since A has size 4×4, T has size $4 \times (4+1) = 4 \times 5$.

2. The equation $A\mathbf{x} = \mathbf{b}$ has a unique solution x. Which combination of MATLAB commands computes x?

When a unique solution exists, the last column of the matrix on reduced row echelon form yields **x**. Hence we can use R=rref(A) to obtain the reduced row echelon form and store it in R. To read the last column we simply write x = R[:,5], because ":"indicates all rows and 5 indicates the fifth (and therefore last) column of R. So we fetch all rows of the fifth column.

Problem 15

To be continued..... Unless I die... or get kidnapped... or get struck by laziness - which is likely.